Performance analysis of atmospheric constrained uncombined multi-GNSS PPP

John Aggrey, Garrett Seepersad and Sunil Bisnath
York University, Toronto, Canada

BIOGRAPHIES

John Aggrey is a Ph.D. candidate at York University, Toronto, Canada in the Department of Earth and Space Science and Engineering. He completed his B.Sc. in Geomatics at the Kwame Nkrumah University of Science and Technology, Ghana, and his M.Sc. at York University. His research currently focuses on the design, development and testing of GNSS PPP software, including functional, stochastic and error mitigation models.

Garrett Seepersad is currently a Ph.D. candidate at York University, Canada researching GNSS PPP and ambiguity resolution in the Department of Earth and Space Science and Engineering. He completed his B.Sc. in Geomatics at the University of West Indies in Trinidad and Tobago. He holds an M.Sc. degree in the same field from York University.

Dr. Sunil Bisnath is an Associate Professor in the Department of Earth and Space Science and Engineering at York University, Toronto, Canada. For over twenty years, he has been actively researching GNSS processing algorithms for positioning and navigation applications.

ABSTRACT

The general objective of this study is to analyze the performance of Precise Point Positioning (PPP) convergence and initialization while stochastically constraining the atmosphere. One specific objective of this study is to review the performance of dual- and triple-frequency PPP solutions using uncombined measurements. The research question to be answered is whether there is any significant benefit in constraining the atmosphere, specifically the ionospheric parameter, in dual- and triple-frequency PPP processing? For thorough performance analysis in the uncombined PPP case, data from 70 globally distributed multi-GNSS stations were processed for the month of February, 2016. The uncombined triple-frequency PPP results with GIM ionospheric constraints showed reduced convergence time compared to unconstrained solutions. At the 68th percentile, a 60% and 78% reduction in convergence was observed for dual- and triple-frequency PPP with ionospheric constraints, respectively. However, more than 50% improvement was observed for constrained triple-frequency uncombined GNSS PPP solution convergence and initialization in the first 5 minutes as compared to dual-frequency PPP solutions. It was concluded that by applying atmospheric constraints in an uncombined multi-GNSS approach, significant improvements were observed for both dual- and triple-frequency PPP implementations.

INTRODUCTION

Conventional GPS and GNSS Precise Point Positioning (PPP) processing makes use of the dual-frequency ionosphere-free linear combination (Zumberge et al. 1997; Héroux et al. 2001; Chen and Gao 2005; Leandro et al. 2011). However, PPP implementation has changed from the usage of dual-frequency measurements to a triple-frequency approach (Hofmann-Wellenhof et al. 2007; Jan 2010; Schönemann et al. 2011; Fairhurst et al. 2001). Opportunities and challenges are both presented with modernized GPS, GLONASS, Galileo and BeiDou constellations when solution accuracy, reliability and integrity become the focus (Henkel and Günther 2010; Elsobeiey 2014). Some prominent areas demanding attention in the quest to enhance PPP performance are convergence and initialization (Seepersad and Bisnath 2012, 2014a, b). Accounting for the challenges of PPP convergence and initialization are key in improving solution quality for various applications. Through linear combinations, previous research contributions have improved the solution quality in dual- and triple-frequency PPP either through linear combinations or by uncombining the raw measurements (Pengfei et al. 2011; Zhang et al. 2013; Odijk et al. 2016; Liu et al. 2017). However, the question of and answer to how close PPP is to Real Time Kinematics (RTK) performance is still blurry. The uncombined PPP approach implies the estimation of ionospheric delay parameters which can further be strengthened through a priori ionospheric knowledge (Collins et al. 2012; Banville et al. 2014; Laurichesse and Blot 2016). The extra widelane provided by third frequency measurements aid in achieving the goal of faster initial PPP solution convergence (Geng and Bock 2013; Li et al. 2013; Tang et al. 2014; Elsobeiey 2014; Laurichesse and Blot 2016; Gayatri et al. 2016).

The positioning performance for low-cost receivers has been shown to improve with Global Ionospheric Maps (GIMs) which are produced by, e.g., the International GNSS Service (IGS). Given that GIM is based on phase-smoothed code observations, the DCB information provided in the IONEX file is only beneficial to code-only, single-frequency receivers. For dual-frequency PPP processing, the significance of GIM in processing is not obvious in the quality of the solution as compared to complete elimination of the ionosphere through linear combination. Using GIM and localized regional ionospheric corrections, performance assessments are provided for dual- and triple-frequency multi-GNSS PPP solutions. Banville et al. (2014) showed that the convergence period of PPP can be reduced with GIM while resolving ambiguities. The level of improvement in convergence seen in the horizontal components was nearly 50% as compared to resolving ambiguities alone without using ionospheric corrections from GIM.

The goal of this study is to answer the question concerning the level of significance of any improvement noticed with atmospheric parameter estimation versus using a priori atmospheric knowledge. Some of the related questions intended to be answered include: (1) Is there any equivalence or differences between combined and uncombined PPP approaches for dual- and triple-frequency measurement processing? (2) Is atmospheric constrained uncombined multi-GNSS PPP nearly comparable to RTK approach? And (3) In terms of solution accuracy and convergence, currently how far away are we from RTK performance?

BRIEF REVIEW OF COMBINED AND UNCOMBINED MULTI-GNSS PPP

PPP processing based on raw observations is gradually becoming the norm as an alternative to iono-free PPP solutions. The advantage it provides includes flexibility in processing current and future GNSS constellations while avoiding noise amplification from linear combinations. The resultant benefit is the ability to extract the ionospheric delays. Using GPS only in PPP processing, the use of raw measurements has been shown to have better performance in positioning and atmospheric modelling (Zhang et al. 2011, 2013). A single-frequency model was also proposed by Shi et al. (2012) to improve the estimation of ionospheric delays in PPP processing. A general GPS/GLONASS/BeiDou/Galileo model was presented by Lou et al. (2016) for PPP single- and dual-frequency processing using raw measurements and using GIM as an a priori constraint. Furthermore, PPP-RTK models which are based on uncombined raw measurements have been analyzed with respect to parameter estimation in a network (Teunissen et al. 2010; Zhang et al. 2011; Odijk et al. 2016). Thus, there is an apparent move towards standardization of the uncombined PPP approach in multi-GNSS processing. However, it must be pointed out that there is limited research regarding this approach and hence it deserves further probing. Shown in equation [1 is the uncombined raw measurement functional model representation.

$$\begin{aligned} p_{r,j}^{s}(i) &= \rho_r^s(i) + dt_r(i) - dt^s(i) + T_r^s(i) + I_{r,j}^s(i) - b_j^s + b_{r,j} + \varepsilon_p(i) \\ \phi_{r,j}^{s}(i) &= \rho_r^s(i) + dt_r(i) - dt^s(i) + T_r^s(i) - I_{r,j}^s(i) - \lambda_j N_{r,j}^s + \varepsilon_\phi(i) \end{aligned} \qquad [1]$$

where $p_{r,j}^{s}(i)$, $\phi_{r,j}^{s}(i)$. denote the pseudorange and carrier-phase observations from satellite (*s*) to receiver (*r*) at epoch (*i*) on frequency *j*; $\rho_r^s(i)$ is the geometric range between the satellite and receiver antennas; $dt_r(i)$ and $dt^s(i)$ represent the receiver and satellite clock errors, respectively; $T_r^s(i)$ refers to the tropospheric delays; $I_{r,j}^s(i)$ is the slant ionospheric delay on GNSS signal propagated at frequency j; b_j^s and $b_{r,j}$ are the satellite and receiver instrumental delays due to the transmitting and receiving hardware, respectively; λ_j is the wavelength on frequency j; $N_{r,j}^s$ is the carrier-phase ambiguity including satellite and receiver phase instrumental delays and initial fractional phase bias; and $\varepsilon_p(i), \varepsilon_\phi(i)$ refer to a combination of observation noise and multipath effect, respectively.

COMBINED AND UNCOMBINED MULTI-GNSS PPP: DUAL- AND TRIPLE-FREQUENCY

As already discussed, the key advantage for uncombining the raw measurements in PPP is to gain access to the ionospheric delay. This distinction is important because it offers an avenue to re-initialize the solution in the event of possible data gaps and cycle-slips, and offers the chance to tighten up the convergence threshold through ionospheric constraining. With respect to satellite geometry, there is no added advantage of the uncombined over the combined approach either in dual- or triple-frequency measurement processing.

Presented in Figure 1 is the horizontal and vertical positioning error components for station NNOR in Australia detailing the similarity in terms of positioning accuracy between the uncombined and combined dual- and triple-frequency measurement processing. The point of how equivalent the two measurement processing approaches are, is further reinforced in

Table 1 with the statistics of the site processing. It can be observed that the difference between the combined and uncombined measurement processing for both dual- and triple-frequency PPP was just units of millimetres. The site NNOR was selected because its results reflect the average results seen for all other stations processed. Post-processed orbits and clock products obtained from the Multi-GNSS Experiment (MGEX) campaign were used (Rizos et al. 2013). In the dual-frequency combined PPP case, the standard L1/L2, E1/E2, B2/B3 ionosphere-free linear combinations were formed for GPS, Galileo and BeiDou satellite systems, respectively. Ionosphere-free linear combinations were formed for L1/L5, E1/E5 and B2/B3 signals in the triple-frequency case for GPS, Galileo and BeiDou satellite systems, respectively. As graphically observed in Figure 1, the combined and uncombined approaches for both dual- and triple-frequency measurements processing are identical. It must be noted that the L1/L2/L5, E1/E2/E5 and B1/B2/B3 biases were not corrected for in the results shown for site NNOR.

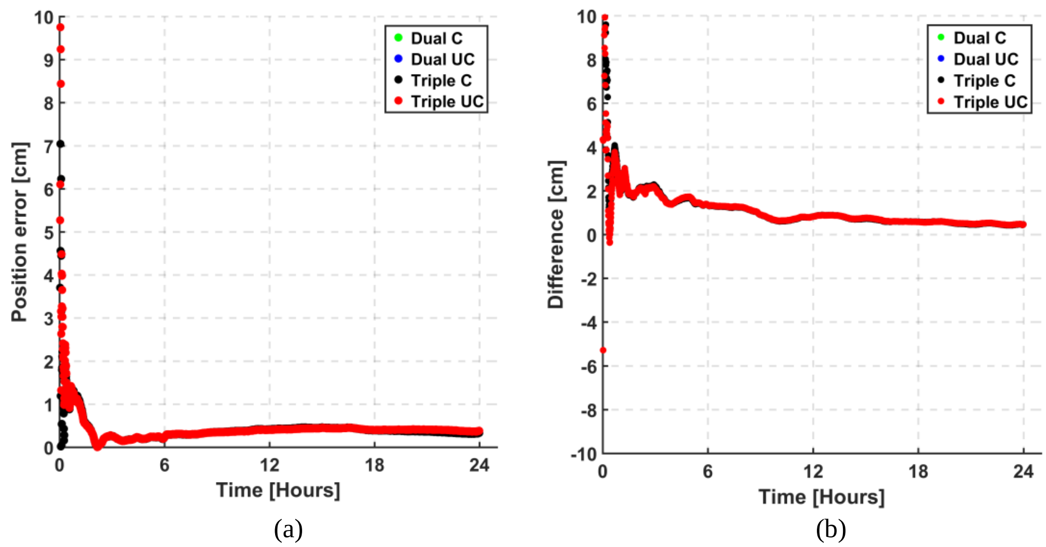

Figure 1: Site NNOR DOY 32 of 2016 located in Australia, illustrating (a) horizontal and (b) vertical components for (1) Dual-frequency combined – "Dual C"; (2) Dual-frequency uncombined – "Dual UC"; (3) Triple-frequency combined – "Triple C"; and (4) Triple-frequency uncombined – Triple UC".

	Combined		Uncombined	
	Dual	**Triple**	**Dual**	**Triple**
Horizontal	9	9	5	5
3D	10	10	7	7

Table 1: Statistics of dual- and triple-frequency float PPP solutions for the site NNOR in 2016 for DOY 32 for both combined and uncombined PPP processing. Results are in mm.

The key point to note is how similar the approaches are in terms of the behaviour of the horizontal and up components. As shown in Figure 1, the combined and uncombined dual-frequency PPP results align well with the triple-frequency combined at the centimetre level of accuracy. This similarity is expected given that both the combined and uncombined are mathematically meant to be produce similar results without the estimation or elimination of additional biases or errors.

Shown in Figure 2 and Table 2 are the residuals for both dual- and triple-frequency PPP float solutions and statistics, respectively, for the site NNOR located in Australia for DOY 32, 2016. Results shown here are meant to be a comparison of both the combined and uncombined approaches in measurement processing.

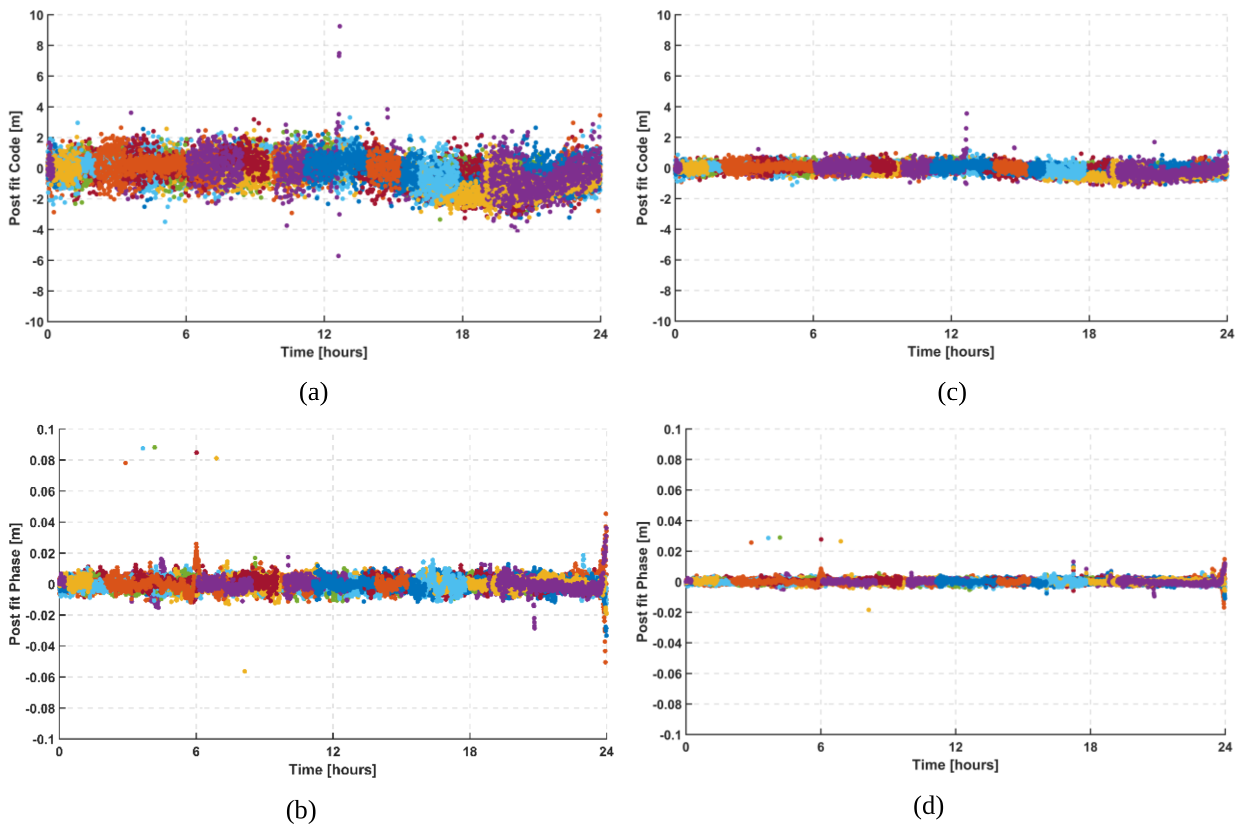

Figure 2: Pseudorange and carrier-phase post-fit residuals for NNOR in 2016 for DOY 32. Results are shown for dual combined (figures 2a, b) and dual uncombined (figures 2c, d).

	Combined		Uncombined	
	Pseudorange	Carrier-phase	Pseudorange	Carrier-phase
RMS	68.8	0.4	22.9	0.2

Table 2: Pseudorange and carrier-phase post-fit residuals for NNOR in 2016 for DOY 32. Results are shown for both combined and uncombined dual-frequency measurement processing. Units are in cm.

As presented in Figure 2, the residual characteristics for dual-combined as well as implied triple-combined measurement processing approaches are similar due to the linear combination of the measurements coupled with the amplification of the noise. Triple-frequency residuals are not shown here because they are similar to the dual-frequency case. Similarly, dual- and triple-uncombined are quite indicative of the benefit of uncombining the raw measurements. The noise in the uncombined residuals as shown in figure 2c and 2d is reduced as compared to the combined approach in figures 2a and b. This is because the formation of linear ionospheric combinations in the combined approach amplifies the noise. However, this is not the case in the uncombined approach given there is no need for linear combinations.

IONOSPHERIC CONSTRAINING WITH GIM

Ionospheric delay models are generated from dual-frequency GNSS observations at ground networks which is beneficial for both ionosphere study and precise GNSS positioning. Using regional or global scales of network stations, ionosphere delay models could be generated which are dependent and correspond to the scope of coverage of the reference networks. GIM is a typical example and is in the form of spherical harmonic functions (Schaer et al. 1998). The assumption made is that the electronic density of the atmosphere is concentrated on an atmospheric layer at a fixed height, usually around 350 km, in the global model recovery. With respect to this assumption, the slant ionospheric delays generated from GNSS observations, are expressed by a combination of the vertical total electronic content (VTEC) and a mapping function. The estimations of the coefficients of the spherical harmonic function are used to represent the VTEC (Schaer et al. 1998). The VTEC is mapped to obtain the slant ionospheric delay through a mapping function after the ionosphere pierce point (IPP).

To investigate the impact of GIM in multi-GNSS PPP processing in both dual- and triple-combined and uncombined approaches, 70 global multi-GNSS stations were selected for processing to find probable answers and arrive at logical conclusions. Figure 3 shows the global distribution of the 70 stations selected for the experiment.

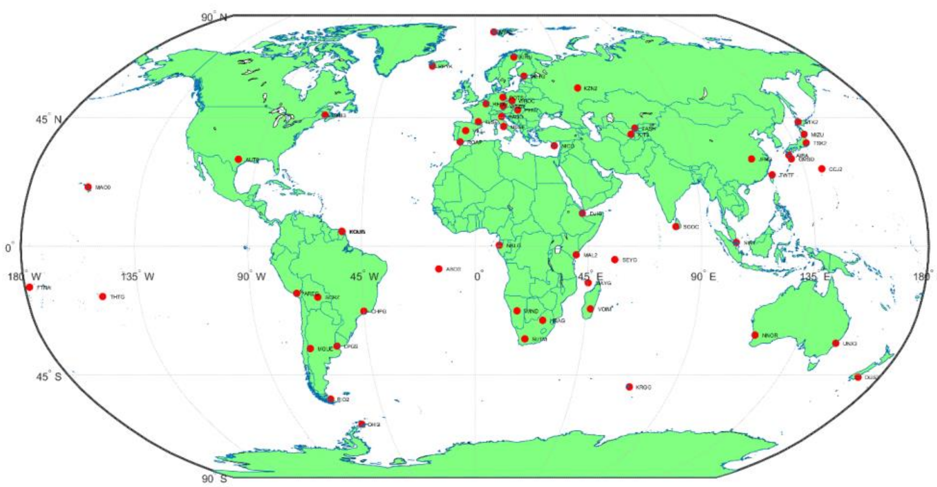

Figure 3: Map of globally distributed stations

Figure 4-7 shows the first hour of 24 hourly horizontal errors for the 70 stations for 4 different processing modes. The solutions presented are based on GPS + GLONASS + Galileo + BeiDou (GREC). The scenarios processed include (a) Dual GREC PPP (b) Dual GIM constrained GREC PPP (c) Triple GREC PPP and (d) Triple GIM constrained GREC PPP. Also shown are the 68th and 95th percentiles for all the processing scenarios. A tight convergence is defined as solutions reaching a horizontal error of 10 cm under 12 minutes, as represented by the black dashed lines.

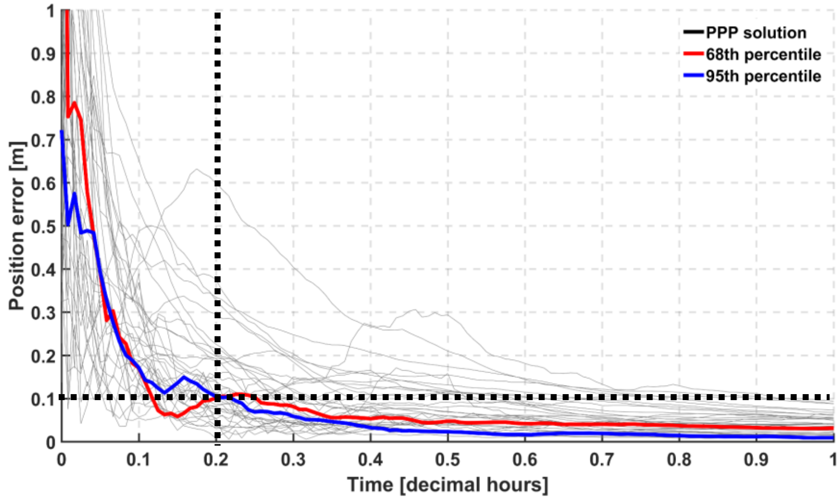

Figure 4: Horizontal positional error (hourly) based on 24 hourly solutions for 70 stations for Dual GREC processing mode.

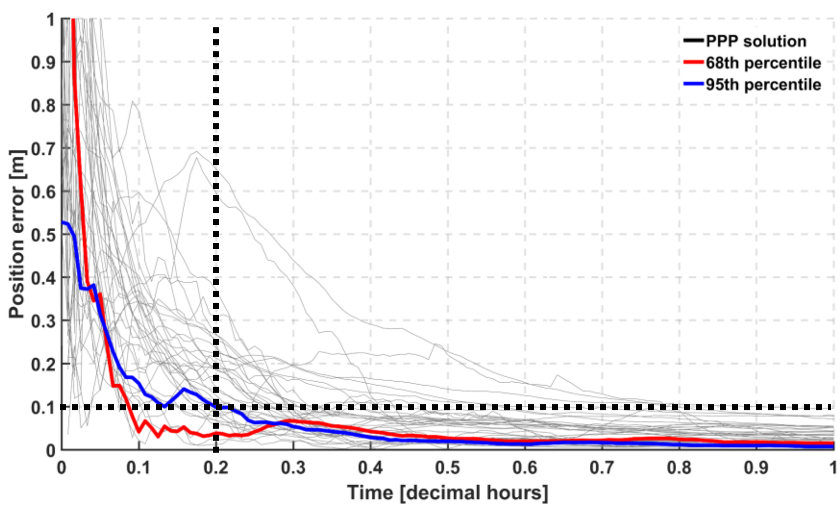

Figure 5: Horizontal positional error (hourly) based on 24 hourly solutions for 70 stations for Dual + GIM GREC processing mode.

Figure 5 and Figure 7 demonstrate how the solutions are affected by the influence of ionospheric constraint application in float solutions. It must be noted that the application of ionospheric constraints using GIM generally helps in the first few epochs by reducing the positional errors. The idea is to fast track convergence and quicker initialization by informing the filter with better slant ionospheric information. However, even with the use of GIM as a priori ionospheric information, the filtered positional estimates in the first few epochs are greatly dependent on the pseudorange measurements, which potentially minimizes the efficacy of GIM. In a float solution case, the solution is helped further on through using multi-GNSS measurements in the processing, which helps tighten convergence as well as quicken PPP initialization.

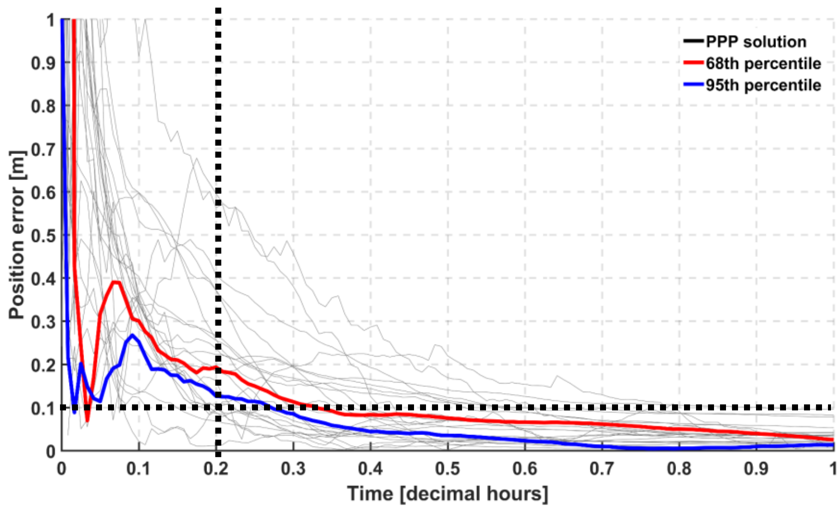

Figure 6: Horizontal positional error (hourly) based on 24 hourly solutions for 70 stations for Triple GREC processing mode.

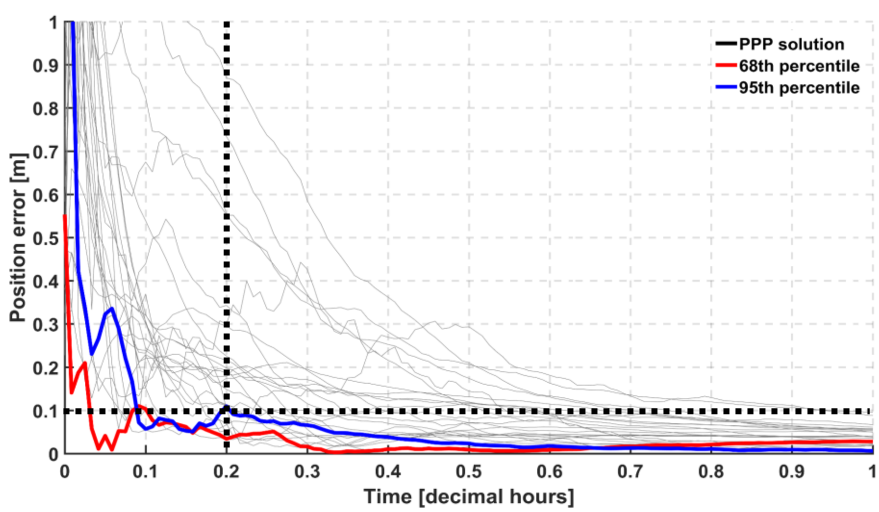

Figure 7: Horizontal positional error (hourly) based on 24 hourly solutions for 70 stations for Triple + GIM GREC processing mode.

The 68th and 95th percentiles for dual and triple GIM constrained GREC PPP showed quicker convergence under 10 cm horizontal error in 12 minutes as compared to dual- and triple-frequency unconstrained PPP. Table 3 shows the convergence times for dual- and triple-frequency PPP processing with and without the application of GIM for the 68th and 95th percentiles of the solutions. Convergence was greatly reduced by 9 and 14 minutes for dual- and triple-frequency PPP solutions with GIM application, respectively.

	Dual	**Triple**
Without GIM	15	18
With GIM	6	4

Table 3: Convergence times for dual- and triple-frequency PPP processing with and without GIM applications for 68th percentile of the solutions. Units are in minutes.

Figure 8, Figure 9 and Figure 10 show the results for dual- and triple-frequency float PPP solutions of the 70 stations with and without the application of GIM ionospheric delay constraining in an uncombined measurement processing mode. GIM was used in providing a priori ionospheric delays to aid in tightening up convergence and for faster initialization. Results shown in figures are for the horizontal components. To show the effect of the constraint, three initialization periods were analysed; 5, 10 and 15 minutes. The criteria for the thresholds were chosen to reflect the convergence of the horizontal components to under 10 cm.

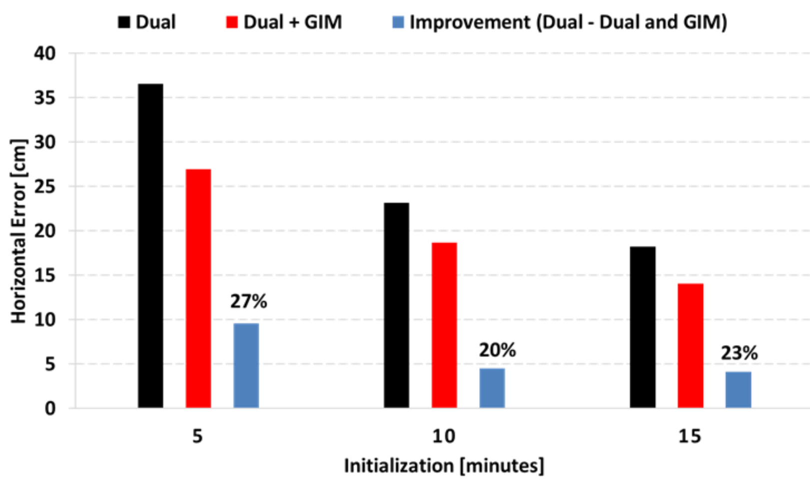

Figure 8: Dual-frequency PPP solutions of 70 stations within a 5, 10 and 15-minute initialization periods with and without GIM constraints. Results for horizontal components are shown.

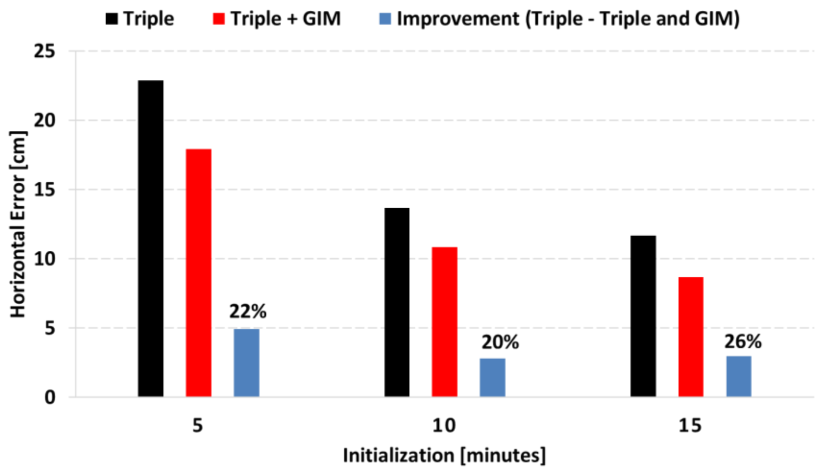

Figure 9: Triple-frequency PPP solutions of 70 stations within a 5, 10 and 15-minute initialization periods with and without GIM constraints. Results for horizontal components are shown.

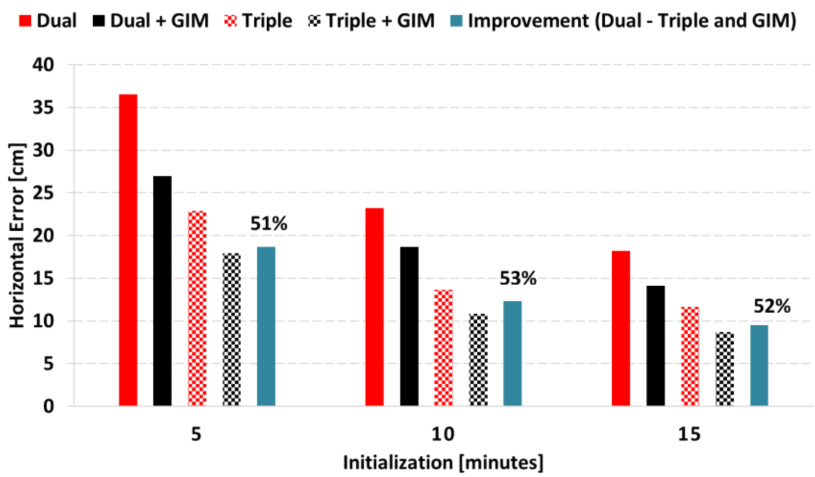

Figure 10: Dual- and triple-frequency PPP solutions of 70 stations within a 5, 10 and 15-minute initialization periods with and without GIM constraints. Results for horizontal components are shown.

The results are indicative of the impact GIM has on ionospheric-constrained PPP solutions. As shown in Figure 8 and Figure 9, there was an average improvement of 27% and 22% when considering initialization periods of 5 minutes. It must be noted that the inherent biases especially for GPS L5 and BeiDou MEO and LEO satellites were not accounted here in order to assess the raw strength of impact of GIM on the solution quality. Considering the poorer quality of BeiDou orbit and clock products, using BeiDou measurements in an uncombined approach does not significantly improve the solution quality. The takeaway is the resultant improvement GIM offers to the PPP initialization as evidenced in Figure 10. A significant 51% improvement is observed for the first 5 minutes for triple-frequency PPP with GIM constraints as compared to dual-frequency PPP. 53% and 52% were also observed for the first 10 and 15 minutes, respectively.

RTK-LIKE PERFORMANCE OF MULTI-GNSS PPP: ARE WE THERE YET?

With the increasing potential of PPP, comparing the technique to RTK only seems logical. As products are getting better and enhanced processing modes are used to provide better float and fixed position estimates, is PPP on the dawn of rubbing shoulders with RTK in the terms of accuracy and convergence? How far away are we currently? Will it always be that PPP will play catch up to RTK or is a time coming when PPP will match RTK millimetre for millimetre? In general, to address the comparison between the two techniques, a series of questions are outlined in an attempt to answer or provoke an answer.

1. ***Where are we now in the quest to RTK-like performance? Are there any significant and specific improvements to be made?***

Figure 11 illustrates the accuracy progression of PPP in different processing modes as compared to RTK and the standard positioning service (SPS). With RTK defining the core of accuracy at the millimetre-centimetre level, the combination of ambiguity resolution and ionospheric constraining draws PPP closer to RTK. PPP has evolved over the years from the conventional float dual-frequency solutions to triple-frequency with AR. It is anticipated that PPP would continue to improve with the mitigation of measurement and hardware biases. So where are we now as a PPP user? It is safe to say that we are in the light green zone getting warmer to the greener turf of RTK performance.

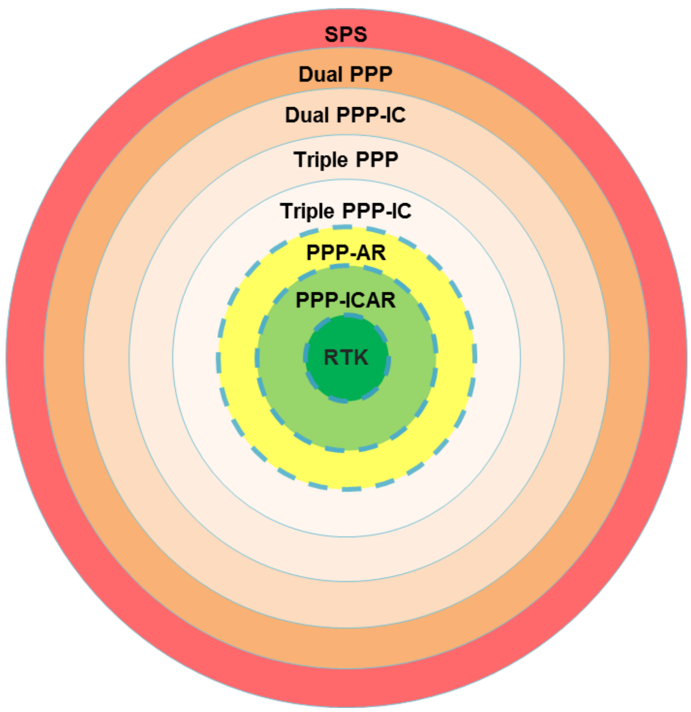

Figure 11: Accuracy hierarchy from RTK to Standard Positioning Service (SPS). This diagram is an augmentation from Collins et al. 2012.

2. ***Does the future support the objective of high accuracy RTK-like PPP performance?***

An optimistic answer would definitely be a yes. However, this response would ignore the significant level of improvement that would be needed to improve PPP solution quality to RTK-like performance. From the issues of slower convergence of PPP to the mitigation of equipment delays, PPP is limited and for that to change would require significant enhancement to PPP algorithms. The added complexity of these enhancements may not necessarily be to the user's advantage. Hence, a more realistic answer would be a maybe with a hint of optimism.

The quest to obtain RTK-like performance with PPP has been on-going for years. Though both techniques give high accuracy solutions, RTK takes the lead in terms of solution stability and convergence making it widely used for many high accuracy applications. PPP is currently on a catch-up mission and it is obvious that the technique is gradually making headway. Though RTK achieves instantaneous convergence through the quick resolution of ambiguities, PPP is continuously breaking grounds in achieving similar results. Are we there yet? Answer is obviously in the negative. However, it must be pointed out by uncombining the raw measurements, either in dual- or triple-frequency measurement processing, access is gained to parameters that aid in re-convergence and further assist in getting better solution quality. PPP still has some limitations that are dependent on the quality of the products being used and error mitigation strategies. Will current centimetre level accuracy solutions from PPP get better? The answer depends on enhancing parameterizations and careful accounting of all potential biases in the solutions. We may not be there yet, but we are bridging the gap a few millimetres at a time.

CONCLUSIONS AND FUTURE RESEARCH

It is concluded that by uncombining and constraining the ionosphere with GIM as a priori information, more than 50% improvement was observed for the first 5, 10 and 15-minute period for triple-frequency PPP in comparison to dual-frequency PPP. This level of improvement is significant for application in which quick convergence is critical. Though results were shown for float solutions, it is expected that by resolving ambiguities, the level of improvement should significantly increase. For future work, it is intended to mitigate time correlated errors to further improve multi-GNSS PPP convergence and initialization while resolving ambiguities. Further investigations would involve accessing the reliability of ionospheric products considering a sparse network of stations.

REFERENCES

Banville S, Collins P, Zhang W, Langley RB (2014) Global and regional ionospheric corrections for faster PPP convergence. Navigation 61:115–124.

Chen K, Gao Y, "Real-Time Precise Point Positioning Using Single Frequency Data," Proceedings of the 18th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS 2005), Long Beach, CA, September 2005, pp. 1514-1523.

Collins P, Lahaye F, Bisnath S (2012) External ionospheric constraints for improved PPP-AR initialisation and a generalised local augmentation concept. In: Proceedings of the 25th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS 2012). pp 3055–3065

Elsobeiey M (2014) Precise Point Positioning using Triple-Frequency GPS Measurements. J Navig 1–13. doi: 10.1017/S0373463314000824

Gayatri A, Reddy PVN, Hazarathaiah A (2016) Handling the Biases for Improved Triple-Frequency Carrier-Phase Ambiguity Resolution PPP Convergence for GNSS. Proceedings of National Conference on Trends in Engineering & Technology - 2K15 (NCTET-2K15), March 15,2016, pp 158 - 165

Geng J, Bock Y (2013) Triple-frequency GPS precise point positioning with rapid ambiguity resolution. J Geod 87:449–460. doi: 10.1007/s00190-013-0619-2

Henkel P, Günther C (2010) Reliable integer ambiguity resolution with multi-frequency code carrier linear combinations. J Glob Position Syst 9:90–103.

Héroux P, Gao Y, Kouba J, et al (2001) Products and applications for Precise Point Positioning-Moving towards real-time. In: Proceedings of the 17th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS 2004). pp 1832–1843

Hofmann-Wellenhof B, Lichtenegger H, Wasle E (2007) GNSS–global navigation satellite systems: GPS, GLONASS, Galileo, and more. Springer Science & Business Media

Jan S-S (2010) Vertical guidance performance analysis of the L1-L5 dual-frequency GPS/WAAS user avionics sensor. Sensors 10:2609–2625.

Laurichesse D, Blot A (2016) Fast PPP Convergence Using Multi-Constellation and Triple-Frequency Ambiguity Resolution. In: Proceedings of the 29th International Technical Meeting of The Satellite Division of the Institute of Navigation (ION GNSS+ 2016), Portland, Oregon, September 2016, pp. 2082-2088.

Leandro RF, Santos MC, Langley RB (2011) Analyzing GNSS data in precise point positioning software. GPS Solut 15:1–13. doi: 10.1007/s10291-010-0173-9

Li J, Yang Y, Xu J, et al (2013) GNSS multi-carrier fast partial ambiguity resolution strategy tested with real BDS/GPS dual- and triple-frequency observations. GPS Solut 19:5–13. doi: 10.1007/s10291-013-0360-6

Liu T, Yuan Y, Zhang B, et al (2017) Multi-GNSS precise point positioning (MGPPP) using raw observations. J Geod 91:253–268. doi: 10.1007/s00190-016-0960-3

Lou Y, Zheng F, Gu S, et al (2016) Multi-GNSS precise point positioning with raw single-frequency and dual-frequency measurement models. GPS Solut 20:849–862. doi: 10.1007/s10291-015-0495-8

Odijk D, Zhang B, Khodabandeh A, et al (2016) On the estimability of parameters in undifferenced, uncombined GNSS network and PPP-RTK user models by means of S -system theory. J Geod 90:15–44. doi: 10.1007/s00190-015-0854-9

Pengfei C, Wei L, Jinzhong B, et al (2011) Performance of Precise Point Positioning (PPP) Based on Uncombined Dual-Frequency GPS Observables. Surv Rev 43:343–350. doi: 10.1179/003962611X13055561708588

Rizos C, Montenbruck O, Weber R, et al (2013) "The IGS MGEX Experiment as a Milestone for a Comprehensive Multi-GNSS Service," Proceedings of the ION 2013 Pacific PNT Meeting, Honolulu, Hawaii, April 2013, pp. 289-295.

Schaer S, Gurtner W, Feltens J (1998) IONEX: The ionosphere map exchange format version 1. In: Proceedings of the IGS AC Workshop, Darmstadt, Germany.

Seepersad G, Bisnath S (2012) Reduction of Precise Point Positioning Convergence Period. York University, Toronto, Ontario, pp 3742–3752

Seepersad G, Bisnath S (2014a) Reduction of PPP convergence period through pseudorange multipath and noise mitigation. GPS Solut 1–11. doi: 10.1007/s10291-014-0395-3

Seepersad G, Bisnath S (2014b) Challenges in Assessing PPP Performance. J Appl Geod 8:205–222.

Shi C, Gu S, Lou Y, Ge M (2012) An improved approach to model ionospheric delays for single-frequency Precise Point Positioning. Adv Space Res 49:1698–1708. doi: 10.1016/j.asr.2012.03.016

Tang W, Deng C, Shi C, Liu J (2014) Triple-frequency carrier ambiguity resolution for Beidou navigation satellite system. GPS Solut 18:335–344. doi: 10.1007/s10291-013-0333-9

Teunissen PJ, Odijk D, Zhang B (2010) PPP-RTK: Results of CORS network-based PPP with integer ambiguity resolution. J Aeronaut Astronaut Aviat Ser A 42:223–230.

Zhang B, Teunissen PJG, Odijk D (2011) A Novel Un-differenced PPP-RTK Concept. J Navig 64:S180–S191. doi: 10.1017/S0373463311000361

Zhang H, Gao Z, Ge M, et al (2013) On the Convergence of Ionospheric Constrained Precise Point Positioning (IC-PPP) Based on Undifferential Uncombined Raw GNSS Observations. Sensors 13:15708–15725. doi: 10.3390/s131115708

Zumberge JF, Heflin MB, Jefferson DC, et al (1997) Precise point positioning for the efficient and robust analysis of GPS data from large networks. J Geophys Res Solid Earth 1978–2012 102:5005–5017.