Do we need ambiguity resolution in multi-GNSS PPP for accuracy or integrity?

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BIOGRAPHIES

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ABSTRACT

With the advent of quad-constellation, triple-frequency and external atmospheric constraints being provided to the PPP user, the novelty and focus of this paper is in the quest to answer the question: Do we really need ambiguity resolution in multi-GNSS PPP for accuracy or for integrity? To address the first component of the question, which is also an area of research that has lacked attention, is an examination of the significance between the float and ambiguity resolved PPP user solution. Is the improvement significant enough for applications such as precision agriculture and autonomous vehicles to justify the additional cost and computational complexity of producing a multi-GNSS PPP-AR solution? Results consist of solution analysis of convergence time (time to a pre-defined performance level), position precision (repeatability), position accuracy (solution error with respect to analysis centre's weekly Site Independent Exchange (SINEX) solution) and residual analysis. Pre-defined user thresholds were selected based on specifications for lane navigation and machine guidance for agriculture. A novel component within the realm of PPP-AR is the analysis of ambiguity resolution as a metric to examine the integrity of the user solution.

The role of ambiguity resolution relies primarily on what are the user specifications. If the user specifications are at the few cm-level, ambiguity resolution is an asset, as it improves convergence and solution stability. Whereas, if the user's specification is at the few dm-level, ambiguity resolution offers limited improvement over the float solution. If the user has the resources to perform ambiguity resolution, even when the specifications are at the few dm-level, it should be utilized. To have a high probability of correctly resolving the integer ambiguities, the residual measurement error should be less than a quarter of a wavelength. Having a successfully resolved and validated solution can indicate to user the solution strength and reliability.

INTRODUCTION

Since the launch of the first Block IIA satellite in 1990, the primary focus of high accuracy positioning has been GPS with dual-frequency measurements. After selective availability was turned off in 2000, it was a natural step to form the ionospheric-free linear combination using GPS data from a single receiver, as some of the early applications were for post-processing of static geodetic data for, e.g., rapid processing of GNSS tracking station data and crustal deformation monitoring (Zumberge et al. 1997). As the popularity of PPP increased, and the advantages such as improved computational efficiency and flexible operating modes without limitation of a localized reference stations, the primary disadvantage of relatively long convergence time to achieve centimetre-level positioning accuracy became more problematic.

The relatively long convergence time fueled research in single receiver ambiguity resolution (AR) (Laurichesse and Mercier 2007; Collins 2008; Mervart et al. 2008; Ge et al. 2008; Teunissen et al. 2010; Bertiger et al. 2010; Geng et al. 2012; Lannes and Prieur 2013). If the ambiguities could be isolated and estimated as integers, then there would be more information that could be exploited to accelerate convergence to provide cm-level horizontal accuracy within an hour of data collection, as illustrated in Figure 1. Resolution of these ambiguities converted the carrier-phases into precise pseudorange



Figure 1: Illustration of the difference between the "float" and "fixed" solution in the horizontal component. NRC1 DOY 179, 2016 located in Ottawa, Canada.

measurements, with measurement noise at the centimetre-to-millimetre level compared to the metre-todecimetre-level of the direct pseudoranges. Collins et al. (2008) and Laurichesse et al. (2009) saw improvements in hourly position estimates by 2 cm and Geng et al. (2010) saw noticeable hourly improvements from 1.5, 3.8 and 2.8 cm to 0.5, 0.5, 1.4 cm for north, east and up, respectively.

By 2010, the advantages of PPP-AR in regards to improved convergence and position stability was well examined but PPP still required over 30 minutes to attain cm-level accuracy (Geng et al. 2010). During this period, research in multi-GNSS (GPS and GLONASS) positioning and estimation of slant ionospheric delay began to exponentially increase. Similar to GPS only PPP-AR, multi-GNSS positioning resulted in improved convergence time and solution accuracy (Cai and Gao 2007, 2013; Banville et al. 2013; Li and Zhang 2014; Aggrey 2015). Li and Zhang (2014) showed a reduction in convergence time from 20 to 11 minutes to attain a predefined threshold of 10 cm 3D. Li and Zhang (2014) and Jokinen et al. (2013) showed the integration of GPS and GLONASS sped up initial convergence and increased the accuracy of float ambiguity estimates, which contributed to enhanced success rates and reliability of fixing GPS ambiguities. Estimation of the slant-ionospheric delay permitted instantaneous convergence if atmospheric corrections were available to the PPP user. Also, if atmospheric corrections are provided, it assists with improving the reliability of ambiguity resolved solution because it will significantly reduce the uncertainties of the ambiguities with a lower frequency by more than one order of magnitude (to ~0.2cy 1σ) (Geng et al. 2010; Collins and Bisnath 2011; Collins et al. 2012; Banville et al. 2014). Naturally, ambiguity resolved triple-frequency was of interest which promised few minutes convergence but also required additional linear combinations to be formed (Geng and Bock 2013). The evolution of the PPP user model is presented in Figure 2 as the performance converges to become more RTK-like.





Over the past decade each of the GNSSs began modernization efforts. The GPS BLOCK IIF is now complete, consisting of 12 satellites transmitting on the L5 band and production of BLOCK III has begun which will have a 4th civilian signal on L1 (L1C) and promises enhanced signal reliability, accuracy, and integrity. For GLONASS, the third generation GLONASS-K satellites will

change from Frequency Division Multiple Access (FDMA) to Code Division Multiple Access (CDMA) which will also transmit five navigation signals on the GLONASS's L1, L2, and L3 bands. The transition from FDMA to CDMA will eliminate the Inter-frequency Channel Biases (ICBs), which will allow GLONASS to be more consistent with other GNSSs, as well as allowing for easier standardization of GLONASS's satellite equipment delay products to enable ambiguity resolution (AR). The European global navigation satellite system, GALILEO is currently under development, with 10 operating satellites and 4 satellites under commission. Lastly, BeiDou begun its transition towards global coverage in 2015. Currently, 5 satellites have been launched and they are currently undergoing in-orbit validation.

It is anticipated that uncombined measurements (no linear combinations) within the PPP mathematical model would become the de facto standard with expanding constellations and frequencies. With the advent of quad-constellation, triple-frequency and external atmospheric constraints being provided to the PPP user, the novelty and focus of this paper is in the quest to answer the question: Do we really need ambiguity resolution in multi-GNSS PPP for accuracy or for integrity?

To address the first component of the question, which is also an area of research that lacked attention, is an examination of the significance of the improvement in accuracy between multi-GNSS float and resolving GPS ambiguities. Is the improvement significant enough for applications such as precision agriculture and autonomous vehicles to justify the additional cost and computational complexity of producing a multi-GNSS PPP-AR solution? Results consist of solution analysis of convergence time (time to a pre-defined performance level), position precision (repeatability), position accuracy (solution error with respect to analysis centre's weekly SINEX solution) and residual analysis. Pre-defined thresholds are based on specifications for lane navigation and machine guidance for agriculture.

A novel component within the realm of PPP-AR is the analysis of ambiguity resolution as a metric to examine the integrity of the user solution. Integrity within the context of the PPP user solution means, the amount of trust that can be placed in the information supplied by the PPP data processing engine. Integrity also relates to the PPP engine's ability to provide timely warnings to users when the solution should not be trusted. Given that in PPP processing all parameters must be accounted for, without multiple solutions (as is in the case with double-differenced static, multi-baseline networks and network RTK) providing integrity information for PPP single receiver estimates is all that more important. Within the context of integrity monitoring, ambiguity resolution will be further examined. Metrics will include: time to first fix, percentage of satellites rejected within partial ambiguity resolution strategy and the reliability of different ambiguity validation techniques.

REVIEW OF UNCOMBINED AND COMBINED MEASUREMENTS IN PPP

The foundation of several multi-GNSS data processing software packages were built around GPS-only ionosphericfree linear combinations which were then expanded to include GLONASS, Galileo and Beidou. Thus, ad hoc linear combinations of measurements were being formed within the PPP user solution, as seen in triple-frequency ambiguity resolution (Geng and Bock 2013). Similar combinations were also formed to allow estimation of the slant ionospheric terms where the Melbourne-Wübbena was decomposed into the narrow lane code (P6) and wide lane phase (L4) to access the slant ionospheric terms (Collins et al. 2012). At this juncture in PPP's history, with multiple constellations, multiple frequencies, AR and instant re-convergence, it was an instinctive step to uncombine the measurements in the user solution, with the advantage of being easily scalable as well as easier access to the estimated slant ionospheric delay. Zhang et al. (2011) was the first to present the uncombined model within PPP, showing "improved performance" as well as and illustrating potentials in tropospheric and ionospheric modeling. While the focus of the paper by Zhang et al. (2011) was on GPS AR, the model presented could have easily been expanded to be multi-constellation as presented in Liu et al. (2017). With interest in ambiguity resolution and slant ionospheric constraining, the mathematical model has been expanded to four combined measurements (p_{IF}^s , p_{NL}^s , Φ_{WL}^s), which is an equivalent number of measurements as the measurements in its natural form (p_1^s , p_2^s , Φ_1^s , Φ_2^s). Within the context of the least-squares estimation and the user solution, there are no benefits of maintaining the combined measurements. The strength of maintaining the measurements in the uncombined form is, the model becomes simpler, more user intuitive and easier to expand to include triple-frequency measurements and additional constellations.

While there have already been several papers that have reviewed uncombined PPP (Zhang et al. 2011, 2013; Odijk et al. 2016; Lou et al. 2016; Liu et al. 2017), there have been some misconceptions. Misconceptions such as, improved accuracy and convergence of uncombined observables in contrast to combined observables within PPP data processing, as well as the practicality of using uncombined PPP in a network solution. The performance of combined and uncombined PPP are equivalent, as the PPP engine is utilizing the same knowledge. To ensure equivalent performance, it is important that the stochastics utilized in combined PPP are also transformed/propagated. In the network solution, while uncombined PPP offers similar advantages of scalability, in the network solution measurement combinations are still required to be re-computed for quality control purposes.

Presented in equation (1) is the mathematical model in matrix notation, which only includes the terms to be estimated, as error sources have been omitted for the sake of brevity.

$\begin{bmatrix} p_1^s \end{bmatrix}$	1	1	1	0	0]	ρ_u^s
$\left \begin{array}{c} p_{2}^{s} \\ p_{2}^{s} \\ \end{array}\right _{-}$	1	1	μ_2	0	0	dt_u^s
$ \Phi_1^s ^=$	1	1	-1	1	0	$I_{u,1}$
$\left\lfloor \Phi_{2}^{s} \right\rfloor$	_1	1	$-\mu_2$	0	1	$\lambda_n N^s$

(1)

 ρ_u^s is the non-dispersive delay between satellite (*s*) and user position (*u*) including geometric delay, tropospheric delay, clock biases and any other delay which affects all the observations identically. $N_{u,i}^s$ is the carrier-phase ambiguity term on the carrier frequencies. The satellite (dt_{IF}^s) and receiver ($dt_{u,IF}$) clock terms presented in equation (2) and (3) represent the transmitter and receiver clock errors which also consists of synchronization errors (equipment delays), which are presented in equation (2) and (3). The receiver can only measure the fractional phase of the first measurement, after which the receiver can keep track of the total phase relative to the initial measurement. The equipment delays are measurement and frequency dependent.

$$dt_{u,IF} = dt_u + d_{u,IF} + \delta_{u,IF} \tag{2}$$

$$dt_{IF}^s = dt^s + d_{IF}^s + \delta_{IF}^s \tag{3}$$

Where $d_{u,IF}^s$ and $\delta_{u,IF}^s$ are the ionospheric-free equipment delays at the satellite and user position.

As is commonly known, when utilizing IGS clocks they are estimated utilizing the ionospheric-free linear combination of the P1 and P2 measurements. As such, when performing single- or dual-frequency processing with uncombined measurements, it is important to also transform the Differential Code Biases (DCBs) to an observable dependent representation. The DCBs utilized in this study were obtained from <u>ftp://ftp.unibe.ch/aiub/CODE</u>. Presented in equation (4) is the transformation of the combined P1P2 DCBs to uncombined representation.

$$\begin{bmatrix} d_1^s \\ d_2^s \end{bmatrix} = \begin{bmatrix} \beta_{IF} \\ -\alpha_{IF} \end{bmatrix} \begin{bmatrix} d_{P1P2_DCB}^s \end{bmatrix}$$
(4)

Presented in equation (5) is the implicit single differenced model utilized within this study. Implicitly differenced observations were adopted because they closer to the physical observables in GPS receivers, and thus are preferable both for aesthetic reasons and because they permit greater insight into their physical and geometrical meaning. The PPP-AR products utilized in equation (6) is provided by CNES. The products consist of wide lane satellite equipment delays (δ_{MW}^s) and the carrier-phase satellite clocks (δt^s). The wide lane satellite equipment delays are daily wide lane pseudorange/carrier-phase equipment delays and the carrier-phase satellite clocks are aligned to the satellite pseudorange clocks within a narrow lane cycle. The alignment of the carrier-phase clocks allows the clocks to be used for the pseudorange and carrier-phase measurements. The IRC products can be downloaded from, https://igsac-cnes.cls.fr/html/products.html and have the prefix "GRG".

$$\begin{bmatrix} p_1^{ps} \\ p_2^{ps} \\ \Phi_1^{ps} \\ \Phi_2^{ps} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & \mu_2 & 0 & 0 \\ 1 & 0 & 1 & 0 & -1 & 1 & 0 \\ 1 & 0 & 1 & 1 & -\mu_2 & 0 & 1 \end{bmatrix} \begin{bmatrix} \rho_u^s \\ dt_u^{ps} \\ \delta_{\Phi_12} \\ I_{u,1}^{ps} \\ \lambda_1 N_{u,1}^{ps} \\ \lambda_2 N_{u,2}^{ps} \end{bmatrix} + \begin{bmatrix} d_1 \\ d_2 \\ \delta_1 \\ \delta_2 \end{bmatrix}$$

where d_1 , d_2 , δ_1 and δ_2 are defined as

$$\begin{bmatrix} d_{1} \\ d_{2} \\ \delta_{1} \\ \delta_{2} \end{bmatrix} = \begin{bmatrix} \beta_{IF} & 0 \\ -\alpha_{IF} & 0 \\ -\beta_{IF} & \frac{f_{2}}{f_{1}} \\ \alpha_{IF} & \frac{f_{1}}{f_{2}} \end{bmatrix} \begin{bmatrix} d_{DCB_{P1P2}}^{s} \\ \delta_{MW}^{s} \end{bmatrix}$$

(6)

PPP INTEGRITY INDICATORS

Adoption of a PPP user model that facilitates ease in scalability in regard to triple frequency measurements and additional constellations is important as a result of all the modernization efforts and growing number of constellations. Also, over the past decade there has been significant research efforts in improving the accuracy of PPP-AR coordinate solutions and the duration of data collection needed to achieve such accuracies. There has been limited work published on the integrity of PPP-AR solution. Integrity is the measure of the trust that can be placed

(5)

in the information supplied by a navigation system. It includes the ability of the system to provide timely warnings to users when the system should not be used for navigation. Given that in PPP-AR processing all parameters have to be accounted for, without multiple solutions as is in the case with double-differenced static, multi-baseline networks and network RTK, providing integrity information for PPP single receiver estimates is all that more important. While it has been illustrated in literature (Seepersad and Bisnath 2015; Teunissen and Khodabandeh 2015) that PPP-AR is equivalent to a double differenced solution using a global network, the solution is more sensitive to localized error sources such as atmospheric error sources and multipath.

Integrity of the user solution is determined by internally examining realistic measurements of solution precision and also by internally detecting and removing of outlier measurements. It is important to have integrity monitoring during data processing as this is the only time when all the information used to form the position solution is present for in depth analysis. In the presented work, PPP integrity indicators include processing filter convergence, parameter estimation covariance and integer-fit residuals. Each is discussed and developed as a means of providing integrity to the PPP solutions. Presented are the different integrity indicators that have been identified and how they are used in PPP. Each shall be expanded in greater detail in the subsequent sections.

Convergence

The use of PPP presents advantages for many applications in terms of operational flexibility and cost-effectiveness. One major limitations is its relatively long initialization time as carrier-phase ambiguities converge to constant values and the solution reaches its optimal precision. PPP convergence depends on a number of factors such as the number and geometry of visible satellites, user environment and dynamics, observation quality, and sampling rate. As these different factors interplay, the period of time required for the solution to reach a pre-defined precision level will vary. Utilizing PPP-AR would accelerate the overall solution convergence to give cm-level horizontal accuracy after 1 hour or less. Collins et al. (2008) and Laurichesse et al. (2009) saw improvements in hourly position estimates by 2 cm horizontal error, compared to 10 cm for the float PPP solution and Geng et al. (2010) saw noticeable hourly improvements from 1.5, 3.8 and 2.8 cm to 0.5, 0.5, 1.4 cm for north, east and up, respectively.



Figure 3: Definition of convergence.

To examine the issue of the user being aware if the solution has truly converged, accuracy specifications of 20 cm and 10 cm was selected to represent the upper and lower bounds of the accuracy specifications for autonomous navigation used in lane navigation (Stefanie Schumann 2014) and machine guidance for agriculture (Wang and Feng 2009). A stringent definition of convergence was established, illustrated in Figure 3, where the solution only attained convergence when it stayed within the accuracy threshold.

Position uncertainty

The weighting of the observations are based on the covariance matrix of the observations, which plays a crucial role in the estimation of the covariance of the parameters. The covariance matrix of the position parameters, also known as the position uncertainty will be discussed and assessed in greater detail to determine its reliability to the PPP user. In most cases, the PPP user has no reference solution available. There have been very few studies that address this aspect of integrity monitoring in PPP to answer the questions: How accurate is my epoch PPP position? And, how realistic is the internal PPP uncertainty estimate? What we are actually asking is how the pseudorange and carrier-phase measurement as well as the modelled errors affect the estimated parameters.

Ambiguity validation

Ambiguity resolution and validation is critical for enabling cm-level accuracy. The integer ambiguity candidates need to be statistically validated before they are accepted as the correct values. Regardless of the ambiguity strategy selected, the most optimal candidates would be determined. The covariance matrix of the real-valued ambiguity parameters can be used as the indicator of the quality of the parameter. Some of the standard ambiguity validation techniques include Probabilistic rounding region, Success-rate, Ambiguity residual test, Fixed solution residual test, F-Ratio test, Difference test, Projector test. Of the different tests, the ratio test, equation (7), is the one of the earliest and most popular test to validate the integer ambiguity. The ratio is formed by the squared norm of the second-best ambiguity residual vector and the squared norm of the best ambiguity residual vector. This ratio is compared against a certain threshold, the critical value and the integer resolved solution is only accepted if the test pasts.

This critical value plays a key role since it is the indicator if the two compared solutions are considered to be discriminated with sufficient confidence. Hofmann-Wellenhof et al. (2007) presented a concise overview of different test statics adopted by different researchers. The choice of the critical value may be regarded as a kind of question mark. Such as, a critical value between 5 and 10 depending on the degrees of freedom and many software such as RTKlib use a fixed critical value, for example, 3. A critical value of 1.5 was adopted based on the recommendation from Han and Rizos (1996) that proposed 1.5 if elevation-dependent weights are used.

$$\frac{(\hat{x}_2 - \bar{x}_2) V_{\hat{x}_1}(\hat{x}_2 - \bar{x}_2)}{(\hat{x}_1 - \bar{x}_1) V_{\hat{x}_1}(\hat{x}_1 - \bar{x}_1)} \ge 1.5$$
(7)

DATASET AND PROCESSING PARAMETERS

To determine the role of ambiguity resolution in multi-GNSS PPP, data from 155 globally distributed stations were processed from DOY 178 to 184 of 2016 provided by IGS which is illustrated in Figure 4. Satellite products provided by Centre National d'Etudes Spatiales (CNES). The data were processed using the York-PPP software (Seepersad 2012; Aggrey 2015). York-PPP was developed based on the processing engine used by the online CSRS-PPP service (NRCan 2013). Dualfrequency receivers tracking either the C/A or





P(Y) - code on L1 were used. For receivers that do not record the P1 observable, the P1C1 code bias correction was applied. A cut-off angle 10° elevation cut-off angle. Slant ionospheric delays and uncalibrated equipment delays were also estimated epoch-by-epoch in the PPP filter. Global Ionospheric Maps (GIMs) produced by the International GNSS Service (IGS) were used as the a-priori estimates to the slant ionospheric term during initialization. The reference stations were analyzed in static mode. Receiver clocks were estimated epoch-by-epoch. The zenith tropospheric delays were also estimated each epoch with a random walk co-efficient of 2 cm/sqrt(hour). The station

coordinates were initialized using a pseudorange only solution with an initial constraint of 10 m. The IGS absolute antenna model file was used and ocean loading coefficients were obtained from Scherneck (2013) for each of the sites processed. In static mode, for ambiguity resolution only candidates with an elevation angle greater than 20° was considered. Modified LAMBDA method (MLAMBDA) was utilized to resolve the ambiguity candidates (Chang et al. 2005).

ASSESSMENT OF THE ROLE OF AMBIGUITY RESOLUTION IN MULTI-GNSS PPP

To answer if we need ambiguity resolution in multi-GNSS PPP for accuracy or for integrity is an intricate one. While commonly known that ambiguity resolution improves solution accuracy and stability, as well as, it is also critical for satisfying user specifications at the few cm-level. Less frequently discussed, is the accuracy specifications are at the few dm-level, such as 10 cm and 20 cm horizontal, what role does ambiguity resolution play?

Convergence

Presented in Figure 5 is the cumulative histogram examining the time required for 50%, 68% and 95% GNSS data to attain a 20 cm horizontal threshold. For the float solution, convergence times of 5, 10 and 40 minutes were required for 50%, 68% and 95% GNSS data to converge. In contrast, the fixed solution required 10 minutes for 50% and 68% of the data to converge and 45 minutes for 95% of the data to converge. No improvements were noted when utilizing the fixed solution at a 20 cm horizontal threshold.



Figure 5: Cumulative histogram illustrating the required convergence of time multi-GNSS data to attain 20 cm horizontal for the float and fixed solutions.



Figure 6: Cumulative histogram illustrating the required convergence of time multi-GNSS data to attain 10 cm horizontal for the float and fixed solutions.

Next, a tighter threshold of 10 cm horizontal was examined, presented in Figure 6. For the float solution to attain a 10 cm horizontal accuracy threshold, 15, 20 and 60 minutes for 50%, 68% and 95%. For the fixed solution, improvements over the float solution becomes more apparent as 10, 15 and 60 minutes is needed for 50%, 68% and 95% of the data to converge. As expected, the role of ambiguity resolution for accuracy becomes more apparent as the threshold is tightened. Improvements in convergence of the fixed solution occurs, because at dm-level position accuracy we are able to successfully resolve the underlying carrier

phase ambiguity term. Also, the rate of convergence of the float PPP solution typically slows down at the few cm to dm-level.

Position uncertainty

Convergence has always been the achilles heel in PPP, which has led to an increase in the reliance on a realistic position uncertainty. The covariance of the estimated position is the main indicator of the solution accuracy, as a reference solution may not always be available. An attempt to address the questions such as how accurate is my epoch PPP position? And how realistic is the internal PPP uncertainty estimate for the float and fixed solution? Integrity was studied by examining the correlation between the determined PPP position error and the position uncertainty scaled to 95%.

The quality of the position uncertainty is defined by rigorous propagation of the observation uncertainties to the estimates of the unknowns. The observations are expected to be normally distributed and uncorrelated. In practice, due to the existence of biases and unknown and/or ignored correlation in the observations, they are not necessarily normally distributed potentially resulting in unrealistic state uncertainty estimates. For single point positioning, the position uncertainty is typically too optimistic. To ensure reliable position uncertainty is provided to the user, it is required that: 1) The stochastic model of the observations is well defined. The covariance matrix must be propagated with realistic observational variances and covariances. And 2) The systematic effects are completely removed (i.e., the functional model is correct). GNSS processing software typically utilizes elevation dependent weights which may be a contributing factor to overly optimistic position uncertainties. Within the PPP code is a module which incorporates the uncertainties in the satellite orbits and clocks from their covariance matrix into the system of the observation equations. Such information will modify the covariance matrix potentially creating a more realistic position uncertainty.

Illustrated in Figure 7 is the correlation plot comparing the average position uncertainty and error for 155 stations in horizontal component. The average position uncertainty as well as the float and fixed position error was taken for epochs at time 1, 5, 10, 15, 20, 25, 30 minutes, 1 to 6, 12, 18 and 24 hours.

For the first hour, the float position uncertainty was overly pessimistic suggesting the error was worse than the true error. For hours 2-6 and 12-18 a strong positive correlation is illustrated such that the average position uncertainty realistically depicts the magnitude of the average error in the component as the solution converged further. While at hours



Figure 7: Solution integrity for the horizontal component.

18-24 the average position uncertainty and errors are correlated, the uncertainty becomes optimistic, suggesting the error is smaller than it actually is. In contrast, the position uncertainty of the fixed solution was overly optimistic, indicating that the error was significantly better than the true error. After 2 hours of processing, the position uncertainty became more realistic in depicting the magnitude of the averaged error.

Ambiguity validation

Integer carrier-phase ambiguity resolution is the key to fast and high-precision. It is the process of resolving the unknown cycle ambiguities of the carrier-phase data as integers. Once successfully resolved, the precise carrier-phase measurements will act similar to pseudorange measurements, thus enabling precise positioning. As previously mentioned, the procedure for carrier-phase ambiguity resolution does not only consist of integer ambiguity estimation, but also includes ambiguity validation testing. Such testing is important, considering the increasing integrity demands on PPP.

Presented in Figure 8 is the site ALGO DOY 178 of 2016 located in Algonquin, Canada, illustrating the differences between the float and fixed solution. The fixed solution consists of the validated solution that passed (accepted) and failed (rejected) the ambiguity validation testing. 81% of the fixed solution passed the validation test and of particular interest is between hours 5.5 and 6 where incorrect ambiguity fixing occurred. The incorrectly fixed solution was correctly identified by ambiguity validation. The sensitivity of ambiguity validation was noted particularly between 15 to 24 hours, where the correctly resolved ambiguity solutions were also rejected.

To better understand the underlying problem, present with ambiguity validation in PPP-AR, the performance was compared to long single relative positioning. baseline Ambiguity validation has been typically described as performing more reliable in relative positioning than in PPP-AR. To compare the performance, relative positioning was used to co-ordinate ALGO and compare the to the PPP performance in Figure 8. For the comparison of relative positioning and PPP-AR, two single baselines were established, 1) ALGO with respect to BAIE with a baseline length of 819 km and 2) ALGO with respect to NRC1 with a baseline length of 199 km. The station distribution is presented in Figure 9. Long baselines were selected to ensure atmospheric errors were not correlated. Canadian Active Control System (CACS) stations ALGO, NRC1 and BAIE were selected to minimize localized effects as these



Figure 8: Examining the reliability of ambiguity validated solution at the site ALGO DOY 178 of 2016 located in Algonquin, Canada. Upper plot illustrates the easting component and the lower plot is the northing component.



Figure 9: Station distribution used to compare the performance of ambiguity validation in single baseline relative positioning and PPP-AR.

are high quality geodetic grade reference stations. For the relative positioning, precise orbits were used and

atmospheric errors were managed similar to PPP-AR. The slant ionospheric term was treated as unknown and the zenith tropospheric delays were estimated each epoch with a random walk co-efficient of 2 cm/sqrt(hour).

Presented in Figure 10 is the comparison between the long single baseline relative positioning solution and PPP-AR. Both techniques were utilized to co-ordinate the station ALGO to examine the performance of ambiguity validation. Sub-plot a) is the horizontal position solution of ALGO with respect to BAIE, sub-plot b) is the ALGO PPP solution and sub-plot c) is ALGO with respect to NRC1. The solutions ALGO-BAIE, ALGO PPP and ALGO-NRC1 had an accepted validated solution of 77%, 81% and 83% respectfully. The relative positioning solutions did not experience similar incorrectly fixed ambiguity solutions between 5.5 and 6 hours as ALGO PPP, indicating improvements in the QC of the PPP engine is needed. Of interest, is the similarities of sensitivity of ambiguity validation of all three solutions between 15 to 24 hours in Figure 10, where the correctly resolved ambiguity solutions were also rejected. These trends suggest that improvements in atmospheric modelling and more realistic stochastic weights needed to ensure more RTK-like performance.



Figure 10: Comparison of the ambiguity validated solution between long single baseline relative positioning and PPP-AR. For relative positioning ALGO was co-ordinated with respect to BAIE with a baseline length of 819 km and with respect to NRC1 199km. GNSS data from DOY 178 of 2016 was used.

Solution statistics, presented in Table 1 were generated by examining each epoch over the 24 hour period, including initial convergence. For all three solutions, ambiguity validation was able to detect initial convergence and identify to the user the float solution was more reliable. The can be seen in the improvement of the summary statistics of the ambiguity resolved solution in contrast to the ambiguity validated solution. Improvements were most notable in the standard deviation where improvements of 4, 4.5 and 2.9 cm were reduced to 0.6, 0.2 and 0.3 for ALGO-BAIE, ALGO PPP and ALGO-NRC1, respectfully.

Table 1: Summary statistics of ambiguity resolved and ambiguity validated solutions for the station ALGO. Statistics compares the performance of ambiguity validation in relative positioning and PPP-AR. GNSS data from DOY 178 of 2016 was used. All units are in cm.

	Ambig	uity resolved	d solution	Ambiguity validated solution			
	st dev	mean	rms error	st dev	mean	rms error	
ALGO-BAIE	4.0	1.7	4.4	0.6	1.4	1.5	
ALGO PPP	4.5	0.7	4.5	0.2	0.4	0.4	
ALGO-NRC1	2.9	1.0	3.1	0.3	0.8	0.9	

CONCLUSIONS

As have been shown in relative positioning and PPP-AR, ambiguity resolution is critical for enabling cm-level positioning. However, what if specifications where at the few dm-level, such as 10 cm and 20 cm horizontal – what role does ambiguity resolution play? To determine the role of ambiguity resolution in PPP, different accuracy specifications and integrity indicators were examined. These indicators include processing filter convergence, parameter estimation covariance, solution position error and ambiguity validation (residual testing).

<u>Convergence</u>: Similar performance was noted between the float and fixed solutions at the 10 and 20 cm horizontal thresholds. As expected, the role of ambiguity resolution for accuracy only become more apparent as the threshold were tightened from the few dm-level to few cm-level. Convergence has always been the Achilles heel in PPP, which has led to an increase in the reliance on a realistic position uncertainty.

<u>Position uncertainty</u>: The covariance of the estimated position is the main indicator of the solution accuracy, as a reference solution may not always be available. Within the first hour, the float position uncertainty was overly pessimistic suggesting the error was worse than the true error. As the solution converged, a strong positive correlation is illustrated such that the average position uncertainty realistically depicts the magnitude of the average error in the component as the solution converged further. While at hours 18-24 the average position uncertainty and errors are correlated, the uncertainty becomes optimistic. In contrast, the position uncertainty of the fixed solution was overly optimistic, indicating that the error was significantly better than the true error. After 2 hours of processing, the position uncertainty became more realistic in depicting the magnitude of the average error.

<u>Ambiguity validation</u>: Ambiguity validation is important, considering the increasing integrity demands on PPP. Of the different tests, the ratio test, was selected as it is the one of the earliest and most popular test to validate the integer ambiguity. For the sites examined, ambiguity validation proved to be a feasible indicator of when a steady state is attained as fixed solutions during initial convergence was rejected. Ambiguity validation in PPP-AR was also compared to single baseline relative positioning. Long baselines were selected to ensure atmospheric errors were not correlated. Sensitivity of ambiguity validation was noted amongst the relative positioning and PPP-AR after 15 hours of processing, where the correctly resolved ambiguity solutions were also rejected. These trends suggest that improvements in atmospheric modelling and more realistic stochastic weights are needed to ensure RTK-like performance.

Do we need ambiguity resolution in multi-GNSS PPP for accuracy or integrity? The answer is presented in Table 2: "it depends". The role of ambiguity resolution relies primarily on what are the user specifications. If the user specifications are at the few cm-level, ambiguity resolution is an asset as it improves convergence and solution stability. Whereas, if the user's specification is at the few dm-level, ambiguity resolution offers limited improvement over the float solution. If the user has the resources to perform ambiguity resolution, even when the specifications

are at the few dm-level, it should be utilized. To have a high probability of correctly resolving the integer ambiguities, the residual measurement error should be less than a quarter of a wavelength (Petovello et al. 2014). Having a successfully resolved and validated solution can indicate to user the solution strength and reliability.



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REFERENCES

- Aggrey JE (2015) Multi-GNSS precise point positioning software architecture and analysis of GLONASS pseudorange biases. York University, Toronto, Ontario
- Banville S, Collins P, Lahaye F (2013) GLONASS ambiguity resolution of mixed receiver types without external calibration. GPS Solut 17:275–282. doi: 10.1007/s10291-013-0319-7
- Banville S, Collins P, Zhang W, Langley RB (2014) Global and regional ionospheric corrections for faster PPP convergence. Navigation 61:115–124.
- Bertiger W, Desai SD, Haines B, et al (2010) Single receiver phase ambiguity resolution with GPS data. J Geod 84:327– 337. doi: 10.1007/s00190-010-0371-9
- Cai C, Gao Y (2007) Precise point positioning using combined GPS and GLONASS observations.
- Cai C, Gao Y (2013) Modeling and assessment of combined GPS/GLONASS precise point positioning. GPS Solut 17:223–236. doi: 10.1007/s10291-012-0273-9
- Chang X-W, Yang X, Zhou T (2005) MLAMBDA: a modified LAMBDA method for integer least-squares estimation. J Geod 79:552–565.
- Collins P (2008) Isolating and estimating undifferenced GPS integer ambiguities. In: Proc. ION NTM. pp 720–732
- Collins P, Bisnath S (2011) Issues in ambiguity resolution for Precise Point Positioning. In: Proceedings of the 24th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS 2011). pp 679–687
- Collins P, Lahaye F, Bisnath S (2012) External ionospheric constraints for improved PPP-AR initialisation and a generalised local augmentation concept. In: Proceedings of the 25th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS 2012). pp 3055–3065
- Ge M, Gendt G, Rothacher M, et al (2008) Resolution of GPS carrier-phase ambiguities in Precise Point Positioning (PPP) with daily observations. J Geod 82:389–399. doi: 10.1007/s00190-007-0187-4

- Geng J, Bock Y (2013) Triple-frequency GPS precise point positioning with rapid ambiguity resolution. J Geod 87:449–460. doi: 10.1007/s00190-013-0619-2
- Geng J, Meng X, Dodson AH, Teferle FN (2010) Integer ambiguity resolution in precise point positioning: method comparison. J Geod 84:569–581. doi: 10.1007/s00190-010-0399-x
- Geng J, Shi C, Ge M, et al (2012) Improving the estimation of fractional-cycle biases for ambiguity resolution in precise point positioning. J Geod 86:579–589.
- Han S, Rizos C (1996) GPS network design and error mitigation for real-time continuous array monitoring systems.
 In: Proceedings of the 9th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GPS 1996), Kansas City, MO, September 1996. pp 1827–1836
- Hofmann-Wellenhof B, Lichtenegger H, Wasle E (2007) GNSS–global navigation satellite systems: GPS, GLONASS, Galileo, and more. Springer Science & Business Media
- Lannes A, Prieur J-L (2013) Calibration of the clock-phase biases of GNSS networks: the closure-ambiguity approach. J Geod 87:709–731. doi: 10.1007/s00190-013-0641-4
- Laurichesse D, Mercier F (2007) Integer Ambiguity Resolution on Undifferenced GPS Phase Measurements and its Application to PPP. In: Proceedings of the 20th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS 2007), Fort Worth, TX, September 2007. pp 839–848
- Laurichesse D, Mercier F, Berthias J-P, et al (2009) Integer ambiguity resolution on undifferenced GPS phase measurements and its application to PPP and satellite precise orbit determination. Navigation 56:135–149.
- Li P, Zhang X (2014) Integrating GPS and GLONASS to accelerate convergence and initialization times of precise point positioning. GPS Solut 18:461–471. doi: 10.1007/s10291-013-0345-5
- Liu T, Yuan Y, Zhang B, et al (2017) Multi-GNSS precise point positioning (MGPPP) using raw observations. J Geod 91:253–268. doi: 10.1007/s00190-016-0960-3
- Lou Y, Zheng F, Gu S, et al (2016) Multi-GNSS precise point positioning with raw single-frequency and dual-frequency measurement models. GPS Solut 20:849–862. doi: 10.1007/s10291-015-0495-8
- Mervart L, Lukes Z, Rocken C, Iwabuchi T (2008) Precise Point Positioning with ambiguity resolution in real-time. In: Proceedings of ION GNSS. pp 397–405
- Odijk D, Zhang B, Khodabandeh A, et al (2016) On the estimability of parameters in undifferenced, uncombined GNSS network and PPP-RTK user models by means of S -system theory. J Geod 90:15–44. doi: 10.1007/s00190-015-0854-9
- Petovello MG, Feng S, Ochieng W (2014) How do you trust centimeter level accuracy positioning? In: GNSS. http://www.insidegnss.com/node/4201.
- Scherneck H (2013) Ocean Tide Loading Provider. http://froste.oso.chalmers.se/loading//index.html. Accessed 2 Jan 2013
- Seepersad G (2012) Reduction of initial convergence period in GPS PPP data processing. York University, Toronto, Ontario

- Seepersad G, Bisnath S (2015) Examining the Interoperability of PPP-AR Products. In: Proceedings of the 28th International Technical Meeting of The Satellite Division of the Institute of Navigation (ION GNSS+ 2015),Tampa, Florida. pp 2845–2857
- Stefanie Schumann (2014) Why we're mapping down to 20cm accuracy on roads. In: HERE 360. http://360.here.com/2014/02/12/why-were-mapping-down-to-20cm-accuracy-on-roads/.
- Teunissen PJ, Odijk D, Zhang B (2010) PPP-RTK: Results of CORS network-based PPP with integer ambiguity resolution. J Aeronaut Astronaut Aviat Ser A 42:223–230.
- Teunissen PJG, Khodabandeh A (2015) Review and principles of PPP-RTK methods. J Geod 89:217–240. doi: 10.1007/s00190-014-0771-3
- Wang J, Feng Y (2009) Integrity determination of RTK solutions in precision farming applications. In: Proceedings of the Surveying and Spatial Sciences Institute Biennial International Conference 2009. Surveying and Spatial Sciences Institute, pp 1277–1291
- Zhang B, Teunissen PJG, Odijk D (2011) A Novel Un-differenced PPP-RTK Concept. J Navig 64:S180–S191. doi: 10.1017/S0373463311000361
- Zhang H, Gao Z, Ge M, et al (2013) On the Convergence of Ionospheric Constrained Precise Point Positioning (IC-PPP) Based on Undifferential Uncombined Raw GNSS Observations. Sensors 13:15708–15725. doi: 10.3390/s131115708
- Zumberge JF, Heflin MB, Jefferson DC, et al (1997) Precise point positioning for the efficient and robust analysis of GPS data from large networks. J Geophys Res Solid Earth 1978–2012 102:5005–5017.